Understanding the Dynamics of Water Use Using Loop Eigenvalue Elasticity Analysis

Sarah E. Drohan[1], Maja Schlüter[2], Steven Lade[2], and Simon A. Levin[1]

[1] Princeton University, Princeton, NJ, USA, [2] Stockholm Resilience Center, Stockholm, Sweden

Summary

As the world population continues to grow the demand on common pool resources like water will continue to increase. In the absence of management, individual extractors may ignore their own impact on supply, leading to a tragedy of the commons. This describes a scenario in which harvesters do not cooperate to extract at a socially optimal level [1, 2, 3].
Tavoni, Schlüter, and Levin [1] examined a socio-ecological ecosystem (SES) model of resource extraction which used ostracism as a mechanism to stabilize cooperation. Much of the SES literature describes qualitatively positive feedback loops as stabilizers of alternative stable states like cooperation.
During my internship, I investigated and applied a tool from systems dynamics, loop eigenvalue elasticity analysis (LEEA), that may allow us to describe quantitatively how the strength of loops impacts the eigenvalues and thus dynamics of an SES. This poster presents the preliminary results of this analysis.

Harvest-Cooperator Model

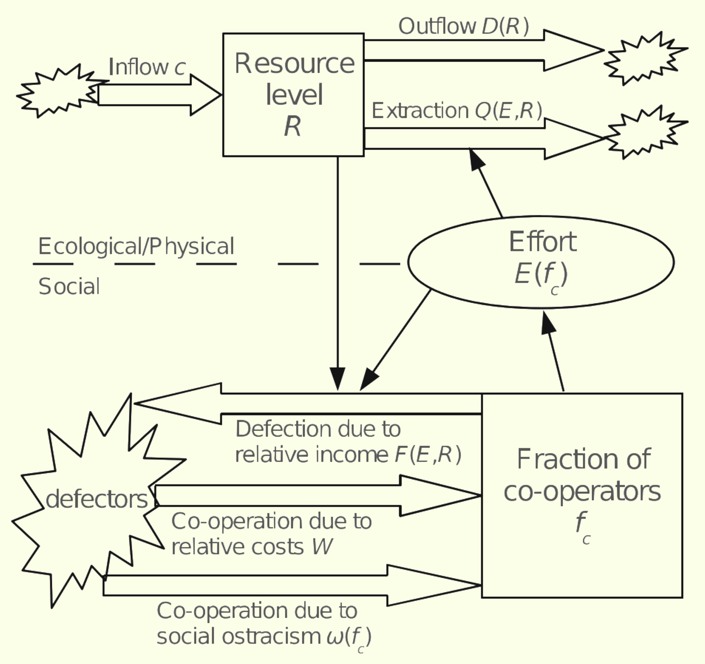

Figure: Diagram from [2]

$$R, f_c = \text{Resource, Fraction Cooperators}$$

$$\frac{\mathrm{d}R}{\mathrm{d}t} = c - d\left(\frac{R}{R_{max}}\right)^k - qER$$

$$\frac{\mathrm{d}f_c}{\mathrm{d}t} = f_c(1-f_c)\frac{\pi_d - \pi_c}{\pi_d}(\omega(f_c) - \pi_d)$$

c = inflow
R_{max} = capacity of resource
d = discharge
k = curvature
q = conversion factor
E = total effort
π_d = payoff to defector
π_c = payoff to cooperator
$\omega(f_c)$ = Gompertz growth function

- This model describes a population of harvesters removing a resource from the environment
- Social ostracism by the cooperators imposes a punishment on defectors that extract more than the socially optimal amount
- Replicator dynamics describe how the fraction of cooperators change over time

Loop Eigenvalue Elasticity Analysis

- Construct a directed graph from the system of differential equations. If variable X appears in the equation for $\frac{dY}{dt}$ then there is an edge from $X \to Y$. The edge gain is defined as $\frac{dY}{dX}$
- Find a shortest independent loop set - the smallest set of loops that are linearly independent
- For each point in time and each loop calculate the loop gain $g = \prod G(e)$ where $G(e)$ is the edge gain for each edge in the loop.
- For each point in time calculate the eigenvalue λ of the linearized system
- The eigenvalue elasticity is defined as $\varepsilon = \frac{d\lambda}{dg}\frac{g}{\lambda}$
- Those loops having a strong influence on the dynamics of the system may have a large corresponding elasticity
- For more information refer to [4, 5]

References

[1] Tavoni A, Schlüter M, Levin S. The survival of the conformist: Social pressure and renewable resource management. Journal of Theoretical Biology. 2012;299:152–161.

[2] Lade SJ, Tavoni A, Levin Sa, Schlüter M. Regime shifts in a social-ecological system. Theoretical Ecology. 2013;6(3):359–372.

[3] Maja Schlüter; Alessandro Tavoni;Simon Levin. Robustness of norm-driven cooperation in the commons. Proceedings of the Royal Society B. 2016;283(1822).

[4] Kampmann CE. Feedback loop gains and system behavior (1996). System Dynamics Review. 2012;28(4):370–395.

[5] Kampmann CE, Oliva R. Loop eigenvalue elasticity analysis: three case studies. System Dynamics Review. 2006;22(2):141–162. Available from: http://doi.wiley.com/10.1002/sdr.333.

Results

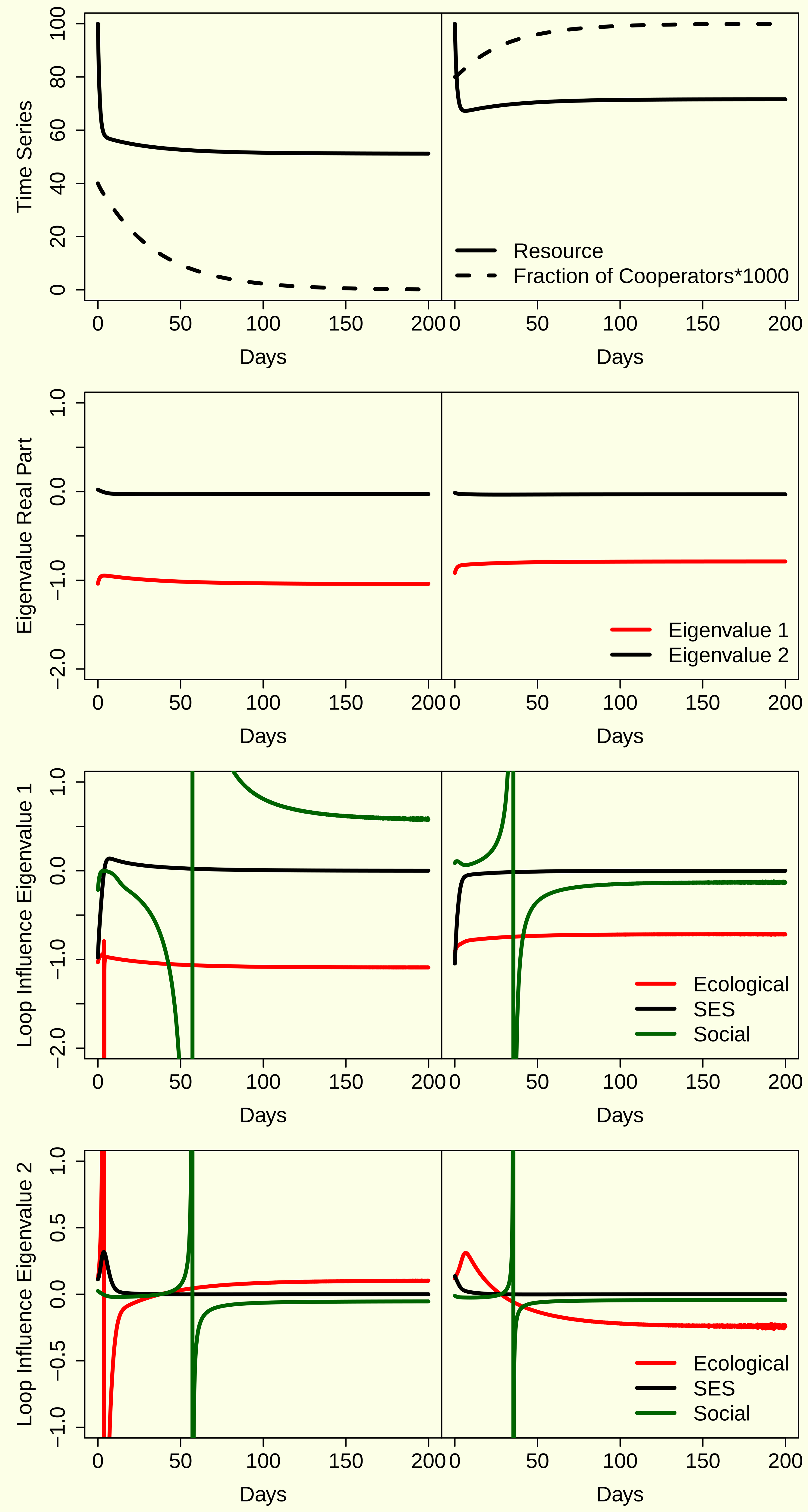

Figure: These plots show the results of LEEA on the harvester SES. Since the original model has two stable equilibria, the analysis is shown on the left for initial conditions leading to the defector equilibrium and on the right for initial conditions leading to the mixed equilibrium. The first and second rows show the time series and eigenvalues of the linearized Jacobian respectively. The third and fourth rows show the loop influence corresponding to each of the eigenvalues where each loop is represented by a different colored line.

- For both initial conditions the interaction loop has little to no influence
- The ecological loop has a negative influence on the first eigenvalue and little influence on the second
- The social loop influence undergoes a discontinuity but has strong influence on the first eigenvalue when converging to the defector equilibrium

Discussion and Next Steps

- The positive feedback loop caused by ostracism in the social loop may be the strongest driver towards the defector equilibrium
- Some of the noise can be attributed to machine precision however the cause of the discontinuity remains unclear
- Can we understand the meaning of loop influence from a dynamical systems theory perspective?
- Use this example to apply LEEA to other SES

Acknowledgements

We are grateful for funding from the NSF Doctoral Training Grant - Mathematical Methods for Water Problems.

sedrohan@princeton.edu