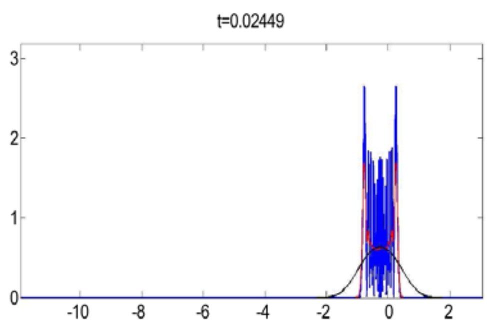
Smoothed Wigner Transforms and Homogenization of Wave Propagation

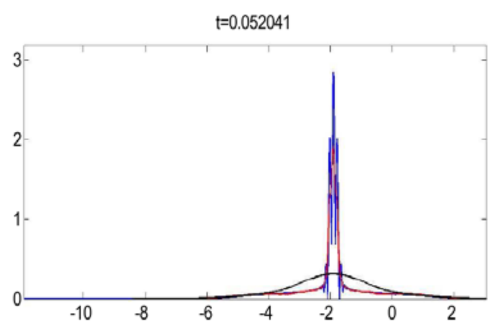
Ph.D. Advisor: Ingrid Daubechies
Career Path: Postdoctoral researcher, Wolfgang Pauli Institute
Contact Information: aathanas@princeton.edu

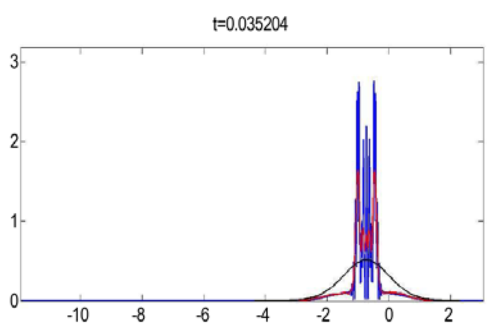
Agissilaos Athanassoulis

Research Program: Wigner transforms (WTs) have been used in many contexts to study phenomena in a phase space setting. In particular, they have been proposed for the study of wave equations. However, because of the WT's highly oscillating 'interference terms', numerical computation using WTs require a very fine discretization, which makes them inefficient. On the other hand, Wigner Measures, developed in part to address this numerical implementation problem, often provide insufficient detail; they are known, e.g., to be unable to recover the spatial structure of caustics.

My thesis proves that smoothed Wigner transforms (SWTs) can be used to study accurately and efficiently the macroscopic scale behavior of solutions to wave equations with multiple scales. After situating the context of the problem, the thesis develops the theoretical framework, deriving, in particular, the evolution equations for SWTs. The new SWT approach is then illustrated by numerical implementations, showing that, in particular, the full macroscopic structure of caustics is captured.







Development and propagation of caustics; full solution (blue); (subcritical) SWT based solution (red); spectrogram based solution (black). The SWT macroscopic solution is significantly faster than the traditional, full (i.e. microscopic) solvers. Spectrogram based solutions in general give poorer results than the SWT, without being significantly faster.