The Preliminary Exam of Xiang Zhou (Jan.20th, 2005)

- Probability Theory by Prof. Cinlar
  - Prove the convergence theorem of positive supermartingale
  - List some martingales related to Weiner process: The third martingale I gave is the exponential martingale \( X_t = \exp(rW_t - \frac{1}{2}r^2t) \). Then he continued to ask me:
  - What’s the limit of \( X_t \) as \( t \) goes to infinity. Why? Is it uniformly integrable? Is it converges in \( L^2 \)?
  - Tell him something about the uniformly integrable?
    In a word, his test was focused on the convergence theory of the martingale. I took his class, probability theory, for preparation. That’s enough, Prof. Cinlar once said to me.

- Numerical Analysis by Prof. Jim Stone
  - How to find a root of a function? I gave bisection, and, Newton method after drawing a picture to illustrate it.
  - Comment on Newton method. Then he gave me the graph of a function (he printed it previously) and asked me what would happen if Newton method is applied.
  - Use forward Euler and backward Euler method to the ODE \( y' = -10y \), and give the condition of stability.
  - Tell him some high-order schemes for ODE. I introduced the idea of linear multistep method and R-K method. He asked me to write a R-K scheme and just tell him the order of accuracy. At that moment, Prof. Weinan asked me about the region of absolute stability for the R-K method I gave.
  - Forward Euler method for the convective equation: \( u_t + au_x = 0 \). Then I gave the Richardson scheme (That was his intention, I think). He asked me about the property of this scheme. – Unstable. The following question was to give the Lax scheme. I did not know which is the Lax scheme he wanted, so I gave both Lax-Friedrich’s scheme and Lax-Wendroff scheme.
Write down the explicit and implicit scheme for the one-dimensional heat equation $u_t = u_{xx}$ and gave the stability condition. Compare the explicit and implicit methods. Then he asked me the coefficient matrix of implicit schemes for one-dimensional and two-dimensional heat equation. Point out the structure of the matrix (band-diagonal).

He asked me lots of questions, but not difficult to answer. Before my preparation for prelim, he gave me three reference books:

2. Richtmyer & Morton, Difference Methods for Initial Value Problems (he said it is old but still very good, but I just took a look because I do not like the old "language". I used Chinese textbooks.)
3. LeVeque, Numerical Methods for Conservation Laws. (That’s my favorite book. Clearly presentation and wonderful examples. Not profound and advanced but it does convey important and vivid ideas.— however, Prof. Jim did not ask me any question about the conservation law.....I spent much time on it...)

PDE by Prof. Weinan E

- Write down the characteristic line of Burgers’ Equation and tell the condition that shocks form.

- When is the constant coefficients one dimensional PDE: $u_t = \sum a_i \frac{\partial u}{\partial x_i}$, well-posed? Actually, he first gave the PDE upto the third derivatives, and then added the fourth derivative. The above is the generalized case (finite term) I wrote down and applied Fourier transform.

- Write down the Green function for the one dimensional problem on $(0,1)$: $-u'' = f(x)$ with Dirichlet boundary condition.

I read the first four chapters of Evans’ book on PDE for this part of my exam.