Questions from Sergey Kryazhimskiy’s preliminary exam (April 26, 2004)

April 28, 2004

Prof. Erhan Çinlar:

1. Give an introductory lecture about martingales (no proofs).

2. Formulate Doob’s stopping time theorem.

3. What condition do we have to impose in order to relax the condition of boundedness of stopping times?

4. Let $X_1, X_2, \ldots, X_n$ be Bernoulli random variables. Prove that, if

$$E \prod_{i=1}^{n} X_i = \prod_{i=1}^{n} E X_i,$$

then the random variables $X_1, X_2, \ldots, X_n$ are independent.

Prof. Simon Levin:

1. All individuals in a population live three years. In the first year they give birth to $a$ offspring in average, in the second year $b$ and in the third year $c$. What is the relationship between $a$, $b$ and $c$, if asymptotically the population doubles every year. Explain the general framework of this problem.

2. What is the equation describing age-structured population with continuous age distribution? How is the equation modified if size structure of the population must be considered as well?
Prof. Philip Holmes:

1. Consider the dynamical system in $\mathbb{R}^2$:

$$\dot{x} = -x - x^2y$$
$$\dot{y} = -y$$

The following reasoning is given to prove that the origin is a globally stable fixed point of this system:

(a) The second equation can be integrated yielding $y(t) = y_0e^{-t}$; hence $y(t) \to 0$ as $t \to \infty$.

(b) Hence, $-x^2y \to 0$ as $t \to 0$.

(c) Hence, for sufficiently large $t$ the first equation is approximately $\dot{x} = -x$, and so $x \to 0$ as well.

Is the reasoning correct? Is the claim correct?

2. Draw the characteristics of the transport equation. Draw a line that is unacceptable for specifying auxiliary condition. Draw a line that is acceptable.