# Rational Inattention: Prices and Information in Macroeconomics

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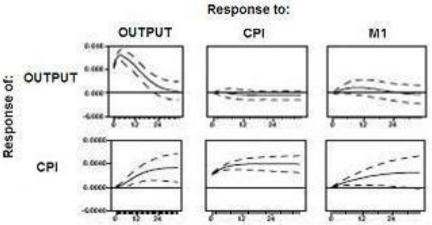
# Outline

1. Motivation
2. Rational Inattention
   - Information Theory
   - Rational Inattention
3. Research Agenda
   - Allocation Problems
   - Pricing game

# Motivation - Sluggishness

- Keynes (1936): Imperfectly adjusting markets (rigid prices) $\longrightarrow$ strong effects of monetary policy
- **Price dynamics is important**. What is the dynamics?
- Everything is sluggish (prices, output, etc.)[1]:




- Stickier prices in:
  - heterogeneous markets
  - markets with few players

- **What micro-foundations for agent's behavior?**
  - Combination of nominal and real sluggishness needed
  - Sluggish information processing?

[1] Sims: Stickiness (1998)

# Motivation - Information Conveyed by Prices

- Hayek (1945): socialism vs capitalism, what's optimal? Local changes $\longrightarrow$ local decisions
  - How to convey information to the man on the spot?
  - **All "relevant" information is concentrated in prices (scarcity, quality, etc.), propagated through markets**
  - No price system - no division of labor...
  - Agents devote limited time to study prices
- Grossman, Stiglitz (70's): quantitative studies of information conveyed through prices, Information paradox

# Motivation - Signal Extraction Introduces Sluggishness

- Lucas(1973): Signal extraction problem - source of shock?
  - Producers decide on output levels, $Y(i) = f(P_R(i)|I_t)$
  - Observe nominal prices $P_N(i) = P_R(i)P$
  - **What is source of shock**: $P_N$ or P?
  - **Monetary expansion can lead to sluggish price level increase and temporal output increase**
- Critics - data on shocks is available very soon
- **Do agents pay attention to it?**

# Motivation Summary - What Next?

- Findings:
  - Keynes: price dynamics is important
  - Hayek: prices convey information
  - Lucas: imperfect inference can lead to sluggishness
- Issues are related - **how to model information** and inference, and their effects **endogenously?**
- Shannon: Information Theory (1948)

# Sims: Rational Inattention (1998, 2003, 2005)

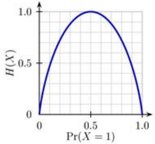
- **Agents modelled as channels of limited information capacity** (no limit to computational ability)
- **Endogenous** nature of **noise** - decide what information to acquire and what to be inattentive to
- **Sluggish responses** - faster to important shocks
- Noisy responses
- Additional appeal: beliefs that agents behave "non-optimally", this provides micro-foundations

# Fundamentals of Information Theory

- Information: flow of reduction in uncertainty about random variable $X$, entropy of $X$:

$$H(X) = -E[\log(p(X))] = -\int p(x)\log(p(x))dx.$$

- Channels: telegraph key, Gaussian, etc.
- Source $(X) \rightarrow$ signal $(Y) \rightarrow$ response $(Z)$




- Reduction of entropy due to observation of signal, mutual information between $X$ and $Y$.

$$I(X;Y) = H(X) - H(X|Y)$$

- **Channel capacity**: maximal rate of transmitting information

$$C(Q) = \max_{P(X)} I(X;Y).$$

- **Coding theorem**: any message whose entropy per time unit is less than capacity can be sent with arbitrary small error
- **Don't need to consider how information is coded and what channel agents uses**
- Data processing theorem: response doesn't convey more than signal

$$I(X;Z) \leq I(X;Y).$$

# ...back to Rational Inattention

- Decision problem is an optimization problem:

$$\max E[U(Z, X, \epsilon)]$$

- **Agents optimize decision strategy** $f(X, Z)$
  - Joint pdf $f(X, Z)$: strategy of information acquisition and reaction to signals
  - Agent chooses response $Z$ to $X$, observed through $Y$
  - Allow for noisy channels: non-unique response of $Z$ to $X$
  - Given prior $g(X) \Rightarrow$ agent chooses conditional $f(Z|X)$
  - Optimal response not conveying too much

$$\max E[U] = \max_f \int f(x, z) U(x, z) dxdz,$$

under constraints:

$$f \geq 0, \qquad \int f(x, z) dz = g(x), \qquad I(X; Z) \leq \kappa.$$

- Alternative view: decide on joint distribution between source and signal and on optimal response to signal

$$\begin{aligned}
\max_{\tilde{z}(Y),p(X,Y)} E[U] &= \max_{\tilde{z},p} \int p(x,y)U(x,\tilde{z}(y))dxdy \\
&= \max_{\tilde{z},p} \int p(x,\tilde{z}^{-1}(z))\tilde{z}'(\tilde{z}^{-1}(z))U(x,c)dxdz \\
&= \max_{f(x,z)} \int f(x,z)U(x,z)dxdz,
\end{aligned}$$

- Shopping - consumption plan vs. decision strategy ?
- Dynamic setting - limited information capacity implies sluggish responses (on average)

$$I(X_t;Y_t) \leq \kappa\Delta t$$

# Research Agenda

- Motivation: RI-General Equilibrium model
- Questions to study:
  - What are differences from complete information solutions?
  - **What does RI imply for market dynamics?**
  - Do prices convey all "relevant" information?
  - What market structures economize on info most?
  - What are efficient numerical methods to be used?
- Setups to study:
  - 2-period consumption problem [2]
  - Social planner, N consumers and N goods
  - Interplay between consumer and seller
  - Fully dynamic consumer's problem

[2] Sims(2005)

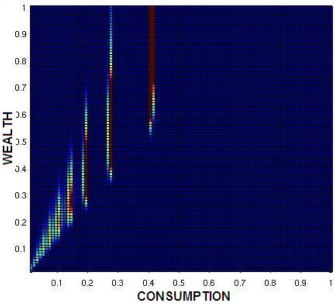
# 2-Period Consumption Problem

- **Imperfectly observed wealth** (checking account)
- Prior on wealth $g(w)$ - needs to ge consistent with strategy $f(w,c)$ - subjective prior, strategy and expectation of utility
- Agent to decide **how much to consume now** $(c)$, what's left is consumed in the next period

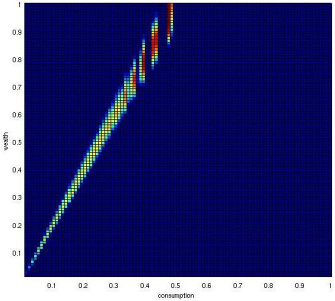
$$\max_{f} \int_{w \geq c} f(w,c)(c^{\theta} + (w-c)^{\theta}) dwdc$$

$$\begin{aligned} f &\geq 0 \\ \int f(w,c)dc &= g(w) \\ I(W;C) &\leq \kappa. \end{aligned}$$

$\kappa =$ 0.88




2.0




- Consistency with prior
- Limited information - imperfect signal on *w*
- Noisy signal is optimal
- More capacity spent on lower *w*'s
- Discrete responses - signal imperfection and concavity of *U*
- Peaks around complete info solution $w = 2c$
- Higher capacity allows for better signal, tighter response

# 2-period problem: Optimization Approaches

- Discretize $W$ and $C$ dimensions
- Finite dimensions - probabilities of discrete states

$$\max_{f_{ij}} \sum_{i \leq j} f_{ij} U(w_j, c_i).$$

- Setup properties
  - Objective function is linear
  - Feasible set is convex, $I(W; C)$: convex function of $f(c|w)$
- Convex high dim. optimization: interior point methods
- Possible to take advantage of FOCs:

$$U_{i,j} - \lambda[\log(f_{ij} - \log(\sum_{k \geq i} f_{ik})] - \omega_j - \mu_{ij} = 0$$

  - Solve for FOCs (assumptions on $\mu_{ij}$)
  - Use as initial condition for simplex method

# Social planner - N consumers, N goods

- Extension of 2-period problem
- Agents endowed with set of goods - quantities not known
- **Social planner decides who to consume what**
- Consumptions of each good have a slack variable - everything that's left is consumed by a given agent

$$\max_f \int f(\vec{e},\vec{c}) \sum_i \Big( \sum_{j \neq i} a_j c_{ij}^{\theta} + a_i (e_i - \sum_k c_{ki})^{\theta} \Big)^{1/\theta} d\vec{c} d\vec{e},$$

constraints:

$$\int f(\vec{e},\vec{c}) d\vec{c} = g(\vec{e}), \qquad I(\vec{E};\vec{C}) \leq \kappa.$$

# Social planner - N consumers, N goods

- Explore what information planner collects
  - sums of endowments, individual endowments, etc.
  - application: property taxes
- Study what is effect of a choice of decision variables
  - what if to decide on specific transfers? might need more capacity - to follow individual endowments
  - application: efficiency of planning bureau vs. market
- Find a way to interpret prices
  - under complete information - prices equal lagrange multipliers on budget constraints
  - shadow costs of constraints can be stochastic
  - look for a way to relate this to decentralized setup
- Develop efficient numerical methods

# Interplay of consumer and producer

- Static market setting: seller and inattentive consumer
- **Nash equilibrium: pricing $p_p(\mu)$ and cons. strategy** $f_c(c|p)$
- Stochastic cost of inputs $\mu \to$ stochastic pricing

  Producer:

$$\forall \mu \qquad p(\mu) = \arg\max_{p} \int (p - \mu) c f_c(c|p) dc,$$

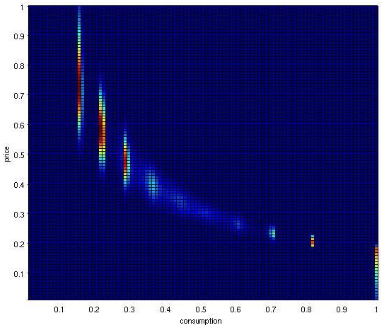
  Consumer:

$$f_c(c|p) = \arg\max_{f(\cdot|\cdot)} \int U(c,p) f(c|p) f(p) dc dp,$$

$$\int f(p,c) dc = f(p), \qquad I(P;C) \leq \kappa.$$

# Interplay of consumer and producer

- Consumer's response to uniform distribution of prices




$\kappa = 1.3$

- Producer's response?
- Discrete consumption implies discrete prices
- Consumer reallocates attention, etc.
- Equilibrium price could be partially discrete
- Price variations bigger than under perfect info

- Find numerical method to solve this setup
- Try to extend it to more agents

# Conclusion

Rational Inattention:

- Treats information endogenously and non-controversially
- Assumes only one free parameter (channel capacity)
- Models what information agents use
- Delivers:
    - Sluggishness observed in data
    - Randomness of responses
    - Dependance of responses on predictability of environment
    - **Microfoundations!**
- **Look for feasible implementations**